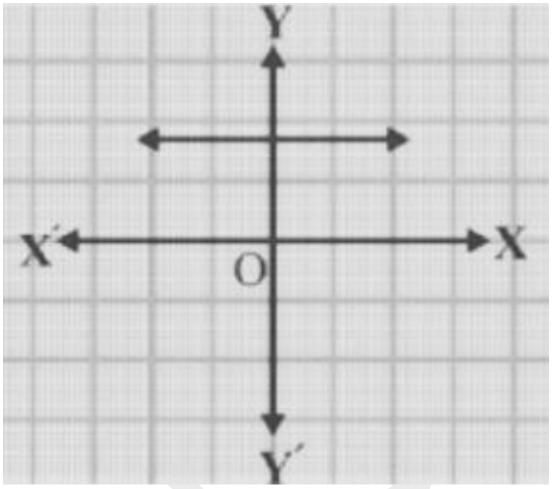
EXERCISE 2.1

1. The graphs of y = P(x) are given in fig. below, for some polynomials P(x). Find the number of zeroes of P(x), in each case.

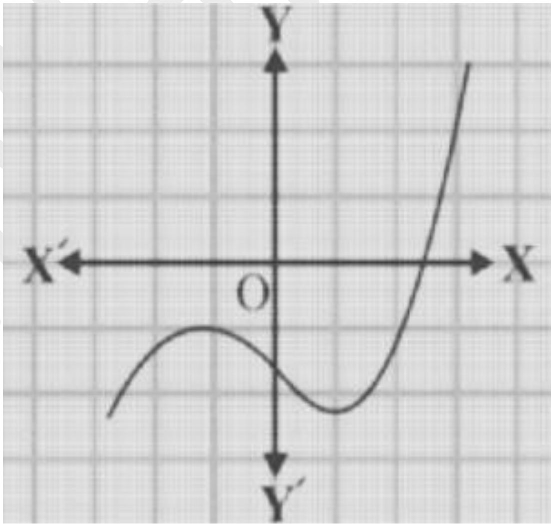
(i) Sol. The graph of given P(x) does not intersect x-axis at any point.

∴ Number of zero = Nil. Ans.



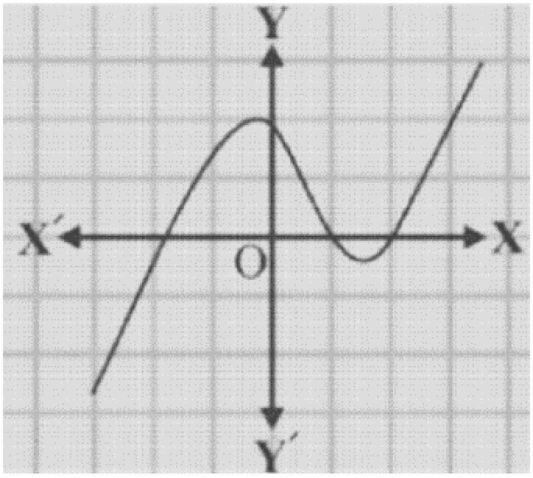
(ii) Sol. The graph of given P(x) intersect the x-axis at one point.

∴ Number of zero = 1. Ans.

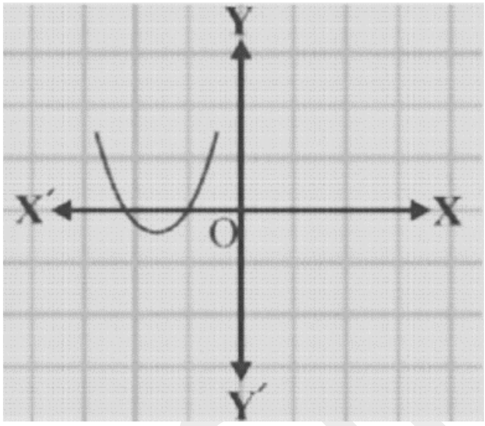


(iii) Sol. The graph of given P(x) intersect the x-axis at three points.

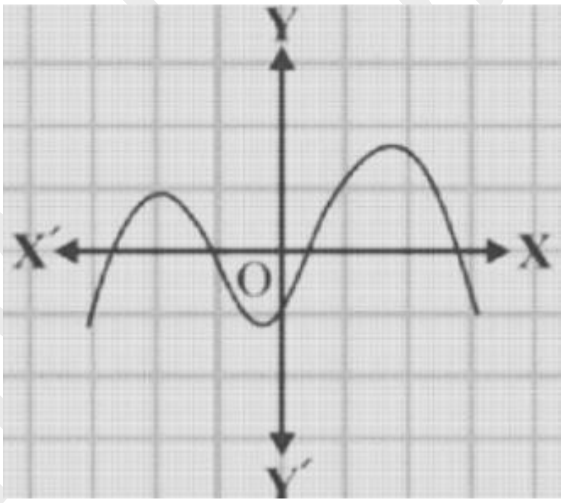
∴ Number of zero = 3. Ans.



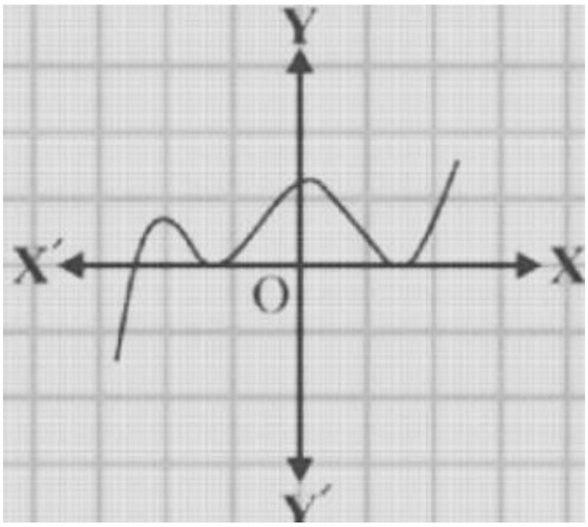
(iv) Sol. The graph of given P(x) intersect the x-axis at two points.
∴ Number of zero = 2. Ans.



(v) Sol. The graph of given P(x) intersect the x-axis at four points.
∴ Number of zero = 4. Ans.



(vi) Sol. The graph of given P(x) intersect the x-axis at three points.
∴ Number of zero = 3. Ans.



EXERCISE 2.1

1. Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.

(i) $x^2 - 2x - 8$

Sol. $P(x) = x^2 - 2x - 8$

$= x^2 - 4x + 2x - 8$

$= x(x - 4) + 2(x - 4)$

$= (x - 4)(x + 2)$

To find the zeroes of P(x) put

$x - 4 = 0$ or $x + 2 = 0$

$x = 4$ or $x = -2$

∴ The zeroes of P(x) are 4 and -2.

∴ Sum of zeroes

$= 4 + (-2)$

$= 2$

$= \frac{-(-2)}{1}$

$= \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$

and Product of zeroes

$= 4 . (-2)$

$= -8$

$= \frac{-8}{1}$

$= \frac{\text{constant term}}{\text{coefficient of } x^2}$

Hence, the relationship between the zeroes and the coefficients is verified.

(ii) $4s^2 - 4s + 1$

Sol. $P(s) = 4s^2 - 4s + 1$

$= 4s^2 - 2s - 2s + 1$

$= 2s(2s - 1) - 1(2s - 1)$

$= (2s - 1)(2s - 1)$

To find the zeroes of P(s) put

$2s - 1 = 0$ or $2s - 1 = 0$

$s = \frac{1}{2}$ or $s = \frac{1}{2}$

∴ The zeroes of P(s) are $\frac{1}{2}$ and $\frac{1}{2}$.

∴ Sum of zeroes

$= \frac{1}{2} + \frac{1}{2}$

$= 1$

$= \frac{-(-4)}{4}$

$= \frac{-\text{coefficient of } s}{\text{coefficient of } s^2}$

and Product of zeroes

$= \frac{1}{2} . \frac{1}{2}$

$= \frac{1}{4}$

$= \frac{\text{constant term}}{\text{coefficient of } s^2}$

Hence, the relationship between the zeroes and the coefficients is verified.

(iii) $6x^2 - 3 - 7x$

Sol. $P(x) = 6x^2 - 3 - 7x$

$= 6x^2 - 7x - 3$

$= 6x^2 - 9x + 2x - 3$

$= 3x(2x - 3) + 1(2x - 3)$

$= (2x - 3)(3x + 1)$

To find the zeroes of P(x) put

$2x - 3 = 0$ or $3x + 1 = 0$

$x = \frac{3}{2}$ or $x = \frac{-1}{3}$

∴ The zeroes of P(x) are $\frac{3}{2}$ and $\frac{-1}{3}$

∴ Sum of zeroes

$= \frac{3}{2} + \left(\frac{-1}{3}\right)$

$= \frac{9-2}{6}$

$= \frac{-(-7)}{6}$

$= \frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$

and Product of zeroes

$= \frac{3}{2} \cdot \left(\frac{-1}{3}\right)$

$= \frac{-1}{2}$

$= \frac{\text{constant term}}{\text{coefficient of } x^2}$

Hence, the relationship between the zeroes and the coefficients is verified.

(iv) $4u^2 + 8u$

Sol. $P(u) = 4u^2 + 8u$

$= 4u(u + 2)$

To find the zeroes of P(u) put

$4u = 0$ or $u + 2 = 0$

$u = 0$ or $u = -2$

∴ The zeroes of P(u) are 0 and -2.

∴ Sum of zeroes

$= 0 + (-2)$

$= -2$

$= \frac{-8}{4}$

$= \frac{-\text{coefficient of } u}{\text{coefficient of } u^2}$

and Product of zeroes

$= 0 \cdot (-2)$

$= 0$

$= \frac{0}{4}$

$= \frac{\text{constant term}}{\text{coefficient of } u^2}$

Hence, the relationship between the zeroes and the coefficients is verified.

(v) $t^2 - 15$

Sol. $P(t) = t^2 - 15$

$= t^2 - (\sqrt{15})^2$

$= (t - \sqrt{15})(t + \sqrt{15})$

To find the zeroes of P(t) put

$t - \sqrt{15} = 0$ or $t + \sqrt{15} = 0$

$t = \sqrt{15}$ or $t = -\sqrt{15}$

∴ The zeroes of P(t) are $\sqrt{15}$ and $-\sqrt{15}$.

∴ Sum of zeroes

$= \sqrt{15} + (-\sqrt{15})$

$= 0$

$= \frac{-0}{1}$

$= \frac{-\text{coefficient of t}}{\text{coefficient of } t^2}$

and Product of zeroes

$= \sqrt{15} \cdot (-\sqrt{15})$

$= -15$

$= \frac{-15}{1}$

$= \frac{\text{constant term}}{\text{coefficient of } t^2}$

Hence, the relationship between the zeroes and the coefficients is verified.

(vi) $3x^2 - x - 4$

Sol. $P(x) = 3x^2 - x - 4$

$= 3x^2 - 4x + 3x - 4$

$= x(3x - 4) + 1(3x - 4)$

$= (3x - 4)(x + 1)$

To find the zeroes of P(x) put

$3x - 4 = 0$ or $x + 1 = 0$

$x = \frac{4}{3}$ or $x = -1$

∴ The zeroes of P(x) are $\frac{4}{3}$ and -1.

∴ Sum of zeroes

$= \frac{4}{3} + (-1)$

$= \frac{1}{3}$

$= \frac{-(-1)}{3}$

$= \frac{-\text{coefficient of x}}{\text{coefficient of } x^2}$

and Product of zeroes

$= \frac{4}{3} \cdot (-1)$

$= \frac{-4}{3}$

$= \frac{\text{constant term}}{\text{coefficient of } x^2}$

Hence, the relationship between the zeroes and the coefficients is verified.

2. Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

(i) $S = \frac{1}{4}$ and $P = -1$

Sol. S = $\frac{1}{4}$ and P = -1

∴ P(x) = x^2 – Sum of zeroes x + Product of zeroes

= $x^2 - \left(\frac{1}{4}\right)x + (-1)$

= $x^2 - \frac{1}{4}x - 1$

∴ Required polynomial can be = $4\left(x^2 - \frac{1}{4}x - 1\right)$

= $4x^2 - x - 4$ Ans.

Alternate method:

Sol. S = $\frac{1}{4}$ and P = -1

⇒ $\frac{-b}{a} = \frac{1}{4}$ and $\frac{c}{a} = -1$

⇒ $\frac{-b}{a} = \frac{1}{4}$ and $\frac{c}{a} = \frac{-4}{4}$

⇒ a = 4, - b = 1 and c = - 4

Or, a = 4, b = -1 and c = - 4

∴ P(x) = $ax^2 + bx + c$

Or, P(x) = $4x^2 - x - 4$ Ans.

(ii) S = $\sqrt{2}$ and P = $\frac{1}{3}$

Sol. S = $\sqrt{2}$ and P = $\frac{1}{3}$

∴ P(x) = x^2 – Sum of zeroes x + Product of zeroes

= $x^2 - \sqrt{2}x + \frac{1}{3}$

∴ Required polynomial can be = $3\left(x^2 - \sqrt{2}x + \frac{1}{3}\right)$

= $3x^2 - \sqrt{2}x - 1$ Ans.

Alternate method:

Sol. S = $\sqrt{2}$ and P = $\frac{1}{3}$

⇒ $\frac{-b}{a} = \frac{\sqrt{2}}{1}$ and $\frac{c}{a} = \frac{1}{3}$

⇒ $\frac{-b}{a} = \frac{3\sqrt{2}}{3}$ and $\frac{c}{a} = \frac{1}{3}$

⇒ a = 3, - b = $3\sqrt{2}$ and c = 1

Or, a = 3, b = $-3\sqrt{2}$ and c = 1

∴ P(x) = $ax^2 + bx + c$

Or, P(x) = $3x^2 - 3\sqrt{2}x + 1$ Ans.

(iii) 0, $\sqrt{5}$

Sol. S = 0 and P = $\sqrt{5}$

∴ P(x) = x^2 – Sum of zeroes x + Product of zeroes

= $x^2 - 0x + \sqrt{5}$

= $x^2 + \sqrt{5}$

∴ Required polynomial is $x^2 + \sqrt{5}$. Ans.

Alternate method:

Sol. S = 0 and P = $\sqrt{5}$

$\Rightarrow \frac{-b}{a} = \frac{0}{1}$ and $\frac{c}{a} = \frac{\sqrt{5}}{1}$

⇒ a = 1, - b = 0 and c = $\sqrt{5}$

Or, a = 1, b = 0 and c = $\sqrt{5}$

∴ P(x) = $x^2 + 0x + \sqrt{5}$

Or, P(x) = $x^2 + \sqrt{5}$ Ans.

(iv) 1, 1

Sol. S = 1 and P = 1

∴ P(x) = x^2 – Sum of zeroes x + Product of zeroes

= $x^2 - 1.\ x + 1$

= $x^2 - x + 1$ Ans.

(v) S = $\frac{-1}{4}$ and P = $\frac{1}{4}$

Sol. S = $\frac{-1}{4}$ and P = $\frac{1}{4}$

∴ P(x) = x^2 – Sum of zeroes x + Product of zeroes

= $x^2 - \left(\frac{-1}{4}\right) x + \frac{1}{4}$

= $x^2 + \frac{1}{4}\ x + \frac{1}{4}$

∴ Required polynomial can be = $4\ (x^2 + \frac{1}{4}\ x + \frac{1}{4})$

= $4x^2 + x + 1$ Ans.

Alternate method:

Sol. S = $\frac{-1}{4}$ and P = $\frac{1}{4}$

$\Rightarrow \frac{-b}{a} = \frac{-1}{4}$ and $\frac{c}{a} = \frac{1}{4}$

⇒ a = 4, - b = −1 and c = 1

Or, a = 4, b = 1 and c = 1

∴ P(x) = $ax^2 + bx + c$

= $4x^2 + 1.x + 1$

= $4x^2 + x + 1$ Ans.

(vi) S = 4, P = 1

Sol. S = 4 and P = 1

∴ P(x) = x^2 – Sum of zeroes x + Product of zeroes

= $x^2 - 4.\ x + 1$

= $x^2 - 4x + 1$ Ans.